Sentimental Habits

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Abstract

Habits and sentiment are key psychological behaviours in asset pricing. This paper studies the interactive impacts of sentiment and habits on asset pricing using the Campbell and Cochrane (1999) habit model as a framework model. A positive sentiment shock emanating from firms is modelled in the drift of the consumption and the habits sensitivity. It has a lagged effect on intertemporal consumption and increases the risk-free rate by an increased habit sensitivity and the precautionary savings motive. The increased habit sensitivity also increases the risk-taking activity of the agents. The model further offers a behavioural explanation of the value premium puzzle in the context of habit models due to relatively lower price-consumption ratios in a negative sentiment environment.

Keywords: Asset Pricing, Behavioural finance, Habit utility models, Market sentiment, Precautionary savings motive

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1 Introduction

Habits and sentiment are important psychological traits that relate to agents' consumption. Habits as defined by psychology are repeated acts of an activity that is developed through reinforcement and repetition. Sentiment on the other hand by the Merriam-Webster dictionary refers to an attitude, thought or judgement prompted by feelings. How does market sentiment which emanates from firms propagate to reinforce or diminish consumption habits? Further, does positive sentiment make people more or less prudent to save or even borrow to maintain their consumption habits? Whilst the individual impacts of sentiment and habits on consumption growth and asset pricing have been studied with many stylised facts established, there are few studies on the interactive effects of market sentiment and habits on asset prices, which I study in this paper.

I use the widely cited Campbell and Cochrane (1999) model, thenceforth the CC model as a framework model to study the interactive habits of sentiment and habits. In the CC model, the surplus ratio is modelled as the consumption relative to the habit level.[1] For a fixed consumption level C_t, if there is a larger surplus in the economy it is because the agents are 'comfortable' with a lower habit level X_t. Conversely, with a small surplus ratio, agents demand a higher habit X_t level they are comfortable with and try to 'attain'. In the field of behavioural psychology, it is known awhile that positive feeling has an effect on decision making, making agents more daring in Isen and Patrick (1983). The agent that has a lower habit level X_t of consumption will thence take less risks. Vice versa, with higher habit expectations, agents are prone to take greater risks to catch up with it much like the *force of habit* in CC aptly titled paper.[2] Given habit levels X_t are unobservable,

[1] With S_t as the surplus ratio and X_t as the habit level.

$$S_t = \frac{C_t - X_t}{C_t} \tag{1}$$

[2] As an analogy, Efing et al. (2015) documented that banking traders take more risks when they are incentivised and used to high bonus payouts. This risk-taking behaviour becomes much lesser with the pay re-structured post the Great Recession.

its effects can only be studied through the observable risk-free rate. There are primarily two avenues for the risk-free rates to be impacted - the intertemporal substitution shift and precautionary savings motive through the surplus (habit) ratio. I investigate how sentiment influences habits through these two avenues.

Firstly, habits have 3 defining characteristics that we are familiar with as emotional beings - the equilibrium habit level (the level of habits we are comfortable with), the persistence of habits (how unwilling we are to change our habits) and the sensitivity (or urge) to regain a comfort habit level when external shocks take place. In the CC model, these are modelled static as $\overline{S}$, [3] ϕ_s and the $\lambda(\cdot)$ respectively. This paper modifies the sensitivity function $\lambda(\cdot)$, the $\overline{S}$ and the consumption growth process with lagged sentiment as an exogenous risk factor, and by restricting some of these modifications, tests how market sentiment impacts habits, and thence the risk-free rate and risk premium.

A lagged market sentiment shock impacts the consumption growth at the longer lag ≈ 6 and the habit sensitivity at a shorter lag of 2. A first finding is a positive sentiment shock from 6 quarters ago in the consumption growth drift tends to decrease consumption level C_{t+1} and increase the risk-free rates.[4] The length of 6 quarters is what it takes for the positive sentiment that emanates from firm-related activity to cascade to impact individual consumption. A positive sentiment shock also increases habit sensitivity to consumption shocks $\lambda(\cdot)$. This increased sensitivity increases the precautionary savings motive and the risk-free rates. The proportion of the risk free rate attributed to the precautionary motive is dependent on the magnitude of the sentiment shock and the surplus ratio. The lagged sentiment shocks however do not make agents more or less persistent in their habits - ϕ_s does not change materially, as the paper shows.

The second finding that this paper seeks to add value is the value premium in habits

[3]The equilibrium surplus ratio $\overline{S}$ has a 1-1 inverse mapping with the equilibrium habit level $\overline{X}$ for a given consumption C_t from the earlier Eq.1.

[4]This negative impact on consumption is counter-intuitive. At the shorter lag, this relationship is positive but insignificant. However the impact turns negative due to the serial correlation of the consumption growth which turns negative at around the longer lag ≈ 6.

models. In Zhang (2005), the value premium has been attributed to two key reasons - costly reversibility and countercyclical price of risk. Companies find it less costly to expand (growth companies) than to wind down capital assets (for value companies). This costly reversibility results in value companies' inflexibility to disinvest in economic downturns when the price of risk is high. On the contrary, surplus ratios are pro-cyclical and mean-reverting (ie. surplus ratios S_t are higher in times of booms than in times of recessions). The mean reversion causes a lower stochastic discount factor placed on the longer-dated cash flows thence generating a higher premium at the longer duration. Growth stocks are known to have longer dated cash flows than value stocks.

It has been documented that habits model do not generate a *value* premium inasmuch as a growth premium in the cross section of stocks as noted in both Santos and Veronesi (2010) and Lettau and Wachter (2007). Santos et al had shown through a simulation exercise that to account for the value premium under the habits model, the value stocks need to exhibit abnormally high cash flow risks, thus creating a *cash-flow risk* puzzle. The Santos et al paper premised a cash flow risk based explanation of the growth-value premium in the CC model. The value premium has similarly been explained through behavioural biases in Barberis et al. (1998). In Barberis et al paper, value stocks are under-weighted due to lower sentiment which subsequently led to their better returns. The lower sentiment directly results from behavioural biases owing to conservatism and evaluation (risk aversion). This impacts the stochastic discount factor rather than cash flow risks. In Santos et al paper, the price-dividend ratio for value stocks had been found to be lower. This is a similar finding in this paper whence using a market-wide sentiment, a price-consumption surface was created with surplus ratio and sentiment as the independent variables. This price-consumption surface tilts lower towards negative sentiment. Under this prevailing negative sentiment, the sentiment improves in a pull to equilibrium leading to higher returns as investors' initial conservative behaviour towards positive earnings stream fades. An advantage of this model postulation is it trivially does not invoke the cash flow risk puzzle, since it explains the value premium through the stochastic discount factor.

In order to model sentiment effects on the equilibrium habit levels and the value premium, the market sentiment from Baker and Wurgler (2007) is used in this paper as an exogenous risk factor. To the author's knowledge, only Sommer (2007) has studied habits and sentiment jointly. Sommer however used an internal habits model and used consumer sentiment to explain the serial correlation in consumption growth generated by habits in a macro-economic approach. This paper is vastly different in that it uses the external habits CC model, considers market rather than consumer sentiment and takes a more asset pricing approach.

The content of the paper is described in the following. In the section 2 that follows, I first discuss a background of the sentiment impacts on asset pricing and the habit utility model. Following this in section 3, I modify the CC model and relate its impact on the risk-free rate in section 3.3. An empirical section on using GMM to examine the performance of the model and imposed restrictions is in section 3.5. I next look at how the models impact the risky assets by calibrating its price consumption plot in section 3.6 and investigating its market prices of risk in section 3.7. A discussion section on the model economic implications at how the models performed historically in section 4.1, the model explanation of irrational exuberance to financial crises and how the model explains the value premium in a behavioural context for habit models in section 4.2. The final section 5 concludes.

2 Background Discussion

I next describe the market sentiment from Baker and Wurgler (2006) and habit utility model used in economics which are the basis of this paper.

2.1 Market Sentiment

The study of sentiment impact on asset pricing has been relatively more recent in literature compared to the habit models. There are three main forms of sentiment in literature - media sentiment, consumer sentiment and market sentiment. The media sentiment refers to sentiment that is obtained from a textual analysis of the media texts in Kelly et al. (2018a) and will not be discussed further. The earlier economic studies look at consumer sentiment which is largely measured through consumer surveys of which the monthly University of Michigan index is the most popular. A macro-economic paper by Carroll et al. (1994) showed that consumer sentiment forecasts household spending. Carroll used the University of Michigan index consumer index and showed it Granger caused the real personal consumption expenditures.[5]

Consumer sentiment indices whilst useful do not reflect market activity and the risk premium, and may be less useful in asset pricing. Later studies evidenced a presence of a market-wide sentiment that is derived from trading activities. These studies include Stambaugh et al. (2012), Kaniel et al. (2008) and Livnat and Petrovits (2009). The DeLong et al. (1990) paper proposed that investors' improved sentiment biased their beliefs about future cash flows and investment risks. Lee et al. (1991) concluded that market-wide sentiment contributes to the differences between prices of closed-end funds and their net asset values. Baker and Wurgler (2007) showed that market sentiment is known to impact the cross section of stocks depending on if their valuation cash flows are highly subjective and difficult to arbitrage. A recent study by Huang et al. (2015) showed that predictive power of sentiment arises from it impacting future cash flows and less so the discount factor. Stambaugh et al. (2012) documented several sentiment anomalies in asset pricing

[5]This consumer sentiment is different from market *firm* sentiment, although household spending still constitutes a large portion - 60% of the US economy. Carroll et al had postulated that consumer sentiment could impact consumers' habits through the precautionary savings motive. Research done by Carroll and Samwick (1995) estimated the precautionary savings motive or prudence to be 40% of liquid assets of households. Later studies by Baiardi et al. (2013) showed a strong precautionary motive to protect against financial and environmental uncertainty to maintain consumption. Pflueger et al. (2017) showed that precautionary savings motive explains about 44% of the variation of the real rate.

and high sentiment with short-sales constraints tend to create mispricing in the short leg of transactions. The papers that have created more often used market sentiment indices are Baker and Wurgler (2006) and the more recent Huang et al. (2015).

In this paper, I use the Baker-Wurgler index due to its longer history. There are two derived sentiment indices in Wurgler '06. Both are derived from a principal component analysis of six proxy sentiment factors - the number and first day returns of IPO stocks, equity share in new issues, NYSE trading volume, dividend premium, and the closed-end fund discount. These are largely firm-related activities and in the economy relates to the supply side of things. One of the Baker-Wurgler indices is further distinguished as residuals from a further regression against macro fundamentals and thence orthogonal against the macro economic fundamentals. This sentiment index is used for this paper study.

One argument of the use of the Baker-Wurgler sentiment index is it constitutes the dividend yield as a component which has been known as a predictor of asset returns. However its overall effect is diluted as one of six factors, and also due to the orthogonal regression. The period of empirical analysis is quarterly from 1965Q3 to 2018Q4, whence the Baker-Wurgler sentiment data is available. The quarterly variable is normalised with its statistical properties shown in table 1. The sentiment residuals are positively skewed and reject the Jarque-Bera normality test.

The sentiment variable x_t evolves according to a AR(1) process with ϕ_x as the persistence coefficient and g_b as a constant. The coefficients are in table 1:

$$x_{t+1} = g_b + \phi_x x_t + \epsilon_{x,t+1} \tag{2}$$

The lagged sentiment shocks $\epsilon_{x,t-n}$ are then used as a state variable on the consumption growth for the models described in section 3.

[Insert Table 1 here.]

2.2 Campbell and Cochrane Habit Utility Model

The economic literature on habit models is vast. There are two broad classes of habit models - internal and external. Internal habit models regards the individual's habits levels as endogenous. External models for example Abel (1990) and Lettau and Uhlig (2000) on the other hand have market-wide exogenous habit levels of consumption the agent seeks to follow in a 'catching up with the Joneses' phenomenon. Another classification of habit models is for habits to be either additive or multiplicative, relative to past consumption. The CC model is an external and additive habits model.

In the CC model, the consumption growth follows an iid process and is replicated here for clarity of exposition:

$$\Delta c_{t+1} = c^a_{t+1} - c^a_t = g_c + \underbrace{\epsilon_{c,t+1}}_{consumption\ shock} \tag{3}$$

The evolving habits X_t are modelled as the surplus ratio S_t which logarithm s_t evolves as a heteroscedastic AR(1) process:

$$s^a_{t+1} = (1-\phi)\bar{s} + \phi s^a_t + \lambda(s_t) \underbrace{(\Delta c_{t+1} - g_c)}_{consumption\ shock} \tag{4}$$

As per convention, small caps are for logarithm values while the large caps indicate the original values. The superscript a indicates aggregate consumption of all agents wherein the habits are external. For notational simplicity, this is omitted in subsequent discussion. With a positive consumption shock (and increased c^a_{t+1}), the third term on the right hand side of the Eq. 4 raises the surplus ratio s_{t+1}, and the next habit level X_{t+1}. The pull of the consumption shock depends on the $\lambda(s_t)$ function in:

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1 - 2(s_t - \bar{s})} - 1 & for\ s_t < s_{max} \\ 0 & for\ s_t \geq s_{max} \end{cases} \tag{5}$$

The $\overline{S}$ (and thence the $\overline{X}$) is defined by:

$$\overline{S} = \sigma_c \sqrt{\frac{\gamma}{1-\phi}} \tag{6}$$

There are 3 key parameters reflecting habit behaviours - $\overline{S}$, ϕ_s and the $\lambda(s_t)$ in the CC model. The $\overline{S}$ reflects inversely the equilibrium habit level $\overline{X}$ which is a consumption level the agent is comfortable with. Suppose $s_t = \overline{s}$ and zero consumption shock in Eq. 4, the next period $s_{t+1} = \overline{s}$ - a level that agents will remain at 'comfortably'. ϕ_s is the habit persistence - how fast does the surplus S_t move back to the equilibrium $\overline{S}$ level. The $\lambda(s_t)$ is the sensitivity of habits to consumption shocks. It makes consumers' behavioural changes fundamentally more important in a recessionary S_t environment with the higher sensitivity value $\lambda(S_t = 0.01)$ than in an expansionary environment with $\lambda(S_t = 0.08)$.

In the original CC model, S_t is calibrated to range from 0 to to 0.097, with the equilibrium value $\overline{S}_t \sim 0.057$. The CC paper used annualised consumption data from 1889 to 1992 with annualised consumption volatility at 1.6%. In this paper, the consumption volatility from 1965Q3 to 2018Q4. The consumption volatility has since been lower at 1.0% resulting in a lower $\overline{S} \sim 0.04$. This lower $\overline{S}$ increases the $\lambda(\cdot)$ sensitivity by Eq. 5. This increased sensitivity occurred in the period 1960s to 2010s where the consumption per capita more than doubled (inflation-adjusted). It appears that increasing wealth makes the consumer more 'sensitive' to consumption shocks.

The table of parameter values used in this paper and the original CC model is in table 5.

[Insert Table 5 here]

3 The Model

3.1 Motivation

I next introduce the primary model in this paper labelled as Model 1. The model follows the standard representative agent economy with utility preferences defined by:

$$u(X_t, C_t) = \delta \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} \tag{7}$$

Its marginal utility of consumption, and second and third order derivatives are respectively:

$$u_c(X_t, C_t) = \delta (C_t - X_t)^{-\gamma} \tag{8}$$

$$u_{cc}(X_t, C_t) = \frac{-\delta\gamma}{(C_t - X_t)^{\gamma+1}} < 0 \tag{9}$$

$$u_{ccc}(X_t, C_t) = \frac{\delta\gamma(1+\gamma)}{(C_t - X_t)^{\gamma+2}} > 0 \tag{10}$$

These utility preference and their derivatives are *the same* as the original CC model. The key manner Model 1 departs from the CC model are its consumption (and dividend) growth processes, the specifications of the $\overline{S}$ and the $\lambda(\cdot)$ all depend exogenously on the Baker-Wurgler market sentiment. I describe these changes and their motivation next.

In the original CC model, the risk-free rate is a constant $\approx 6.2\%$ with the intertemporal rate of substitution and the precautionary measure offsetting each other exactly.[6] This was before the mid 1990s when the model was developed. However since the 2000s, quantitative easing has made this a tight restriction. A graph of the risk-free rates proxied by 3 month Treasury bills from 1965Q3 to 2018Q4 is in figure 3 showing it to be non-

[6] The risk-free rate in the CC model is given by:

$$r_f = -log(\delta) + \gamma g - \frac{\gamma}{2}(1-\phi) \qquad for\ CC\ model \tag{11}$$

constant over the period.

[Insert Figure 3 here.]

Wachter (2006) further modified the CC model for the risk-free rate to depend on the surplus ratio endogenously by modifying the equilibrium $\overline{S}$. I do so in this paper by having the firm market sentiment as an exogenous risk factor that drives both the consumption growth and the agent's habit behaviours. The use of the market sentiment as a risk factor is useful since when marginal utility is high in recession states, negative sentiment results in a high risk premium and a lower discount rate factor. In economic boom times vice versa, positive sentiment contributes to a lower risk premium and a higher discount rate factor. This sentiment effect on the risk premium in the different economic states is in the *same direction* as the endogenous surplus ratio. The surplus ratio and the market sentiment have a richer interaction effect on the risk-free rate as later discussed.

3.2 Consumption and Dividend Growth Processes

At this stage, I discuss the type of economic equilibrium for the model, and how market (firm) sentiment factors into the consumption growth. An endowment Lucas-type economy is assumed for which production and consumption are exogenous processes. Whilst exogenous, consumption growth[7] is however linked to production through the market sentiment in:

$$\Delta c_{t+1} = g_c + \underbrace{\alpha_c \epsilon_{x,t-n}}_{\textit{lagged sentiment shock}} + \underbrace{\epsilon_{c,t+1}}_{\textit{consumption shock}} \tag{12}$$

The consumption growth is regressed against various n lags of sentiment shocks $\epsilon_{x,t-n}$ with the results in table 3. The quarterly data from 1965Q3 to 2018Q4 is used to minimise measurement errors from monthly data. A lag of $n = 6$ quarters is found to have the best fit R^2 with $\alpha_c \approx -0.0027 < 0$.

[7]The consumption data includes contributions from the non-durables and the services only.

[Insert Table 3 here]

The surplus ratio in Model 1 is similar to the AR(1) process in the CC model Eq.4 except with the additional specification of the $\epsilon_{x,t-n}$

$$s^a_{t+1} = (1-\phi)\bar{s} + \phi s^a_t + \lambda(s_t)(\Delta c_{t+1} - g_c - \alpha_c \epsilon_{x,t-n}) \tag{13}$$

This sentiment from Baker-Wurgler described in section 2.1 is largely firm-related and arises from production. This explains the length of the time lag = 6 as the firm related activity cascades to impact consumption.[8] There are possibly two ways for the firm sentiment effects to filter through - the labour market and the capital investment. As firm sentiment improves, this leads to more hiring and also capital investment, which eventually increases consumption as spending increases. It is counter-intuitive for $\alpha_c < 0$. A positive $\epsilon_{x,t+n}$ shock would be expected to raise consumption and less savings at $t+n$. This applies only for the shorter lags $n = 1$ *or* 2 as observed in the table 3 and the positive contemporaneous correlation in table 2, although the relationship is weak and insignificant. At the longer lag, due to the serial correlation of the consumption growth[9], this effect turn negative.

The dividend growth is modelled similarly as the consumption growth driven by a sentiment shock x_{t-n} with a different sensitivity α_d:

$$\Delta d_{t+1} = g_d + \underbrace{\alpha_d \epsilon_{x,t-n}}_{\textit{lagged sentiment shock}} + \underbrace{\epsilon_{d,t+1}}_{\textit{Dividend shock}} \tag{14}$$

The error $\epsilon_{d,t} \sim N(0, \sigma^2_d)$ reflects news shocks that impact the economic fundamentals and the dividend shocks. A series of regression of the dividend growth against the lagged

[8] In order to test this, a separate lagged regression is performed of consumption against the University of Michigan *consumer* sentiment index. A smaller lag (of $n = 0$ or ≈ 1) of greater significance is observed for this consumer sentiment index highlighting the differential impacts of these sentiment indices.

[9] The correlogram in figure 2 shows a zero or even negative autocorrelation at the higher lags 4 to 6. This is documented in papers Heaton (1995) and Ferson and Harvey (1992) which explain the reasons for this serial correlation

sentiment shocks is also performed to determine the order of lag n with the results in table 3. A lag of $n = 6$ quarters is similarly found to have the best fit R^2 for the dividend regression with $\alpha_d >> \alpha_c$ as the market sentiment has a greater direct impact on dividend flows then consumption.

The sentiment shocks $\epsilon_{x,t-n}$ are known at time t from Eq. 2. The addition of the sentiment shocks reduces the annualised volatility of the consumption residuals $\epsilon_{c,t}$ marginally from 1.0% to $\sim$ 0.098% in Eq. 3. This resonates with Bansal and Yaron (2004) of a small predictable component in the drift. This 'predictability' in the model is attributed to cascade effects from the firm-related activity on consumption.

A shortcoming of the original CC model is that both the dividend growth and the consumption growth are driven by a fixed correlation of their Brownian motions.[10] In the long term this causes both of them to behave similarly from each other. The use of market sentiment as an exogenous state variable adds richness to the co-movements of the dividend and consumption growth compared to the CC model. Consumer and corporate reactions to macro-economic changes are different though. In this model, the dividend growth and the consumption growth are co-integrated processes through a common multiple of the α's in both processes with the assumption of stationarity and mean-reversion of the sentiment Eq. 2. The correlation between the Δd_t and Δc_t is non-constant across time horizon and depends on the lagged sentiment shocks. The long-run correlations amongst the consumption shocks $\epsilon_{c,t}$, dividend shocks $\epsilon_{d,t}$ and sentiment shocks $\epsilon_{x,t}$ computed from 1965Q3 to 2018Q4 are in table 2.

[Insert Table 2 here.]

In summary, the motivation for the proposed model 3 discussed in this section is to mitigate the constant risk-free rate and the fixed correlation between dividend and consumption growths of the CC model.

[10] $\mathbb{E}_t[\epsilon_{d,t}, \epsilon_{c,t}] = \rho dt$

3.3 Risk Free Rate

I next consider the specification of the risk-free rate for the Model 1 from the stochastic discount factor M_t. Following standard derivation, this stochastic discount factor or the marginal rate of substitution from the ratio of the marginal utility at time t to time $t+1$ is:

$$M_{t+1} = \delta \frac{u_c(C_{t+1}, X_{t+1})}{u_c(C_t, X_t)} = \delta \left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} \tag{15}$$

The substitution of Eqs. 12 and 13 for the consumption growth and the surplus equation results (with the derivation in the appendix 5.0.1) results in:

$$M_{t+1}|_t = \delta e^{-\gamma[g_c + \alpha_c \epsilon_{x,t-n} + (1-\phi_s)(\bar{s}-s_t) + (1+\lambda(s_t))\epsilon_{c,t}]} \tag{16}$$

In the literature, the risk-free rate is driven primarily by the intertemporal rate of substitution and the precautionary savings motive. This motive has always been studied with an uncertain factor. The uncertain factor in this paper is future consumption in relation to habits that the consumer takes precautionary measures to sustain. Consumption growth in itself would have been too 'smooth' to hedge against. A high surplus ratio makes the agent less prudent and save less, while a low surplus ratio makes the agent more prudent and save more. This can increase savings rate in recessionary times, when spending and consumption are most needed. This expression of the prudence directly in terms of the surplus ratio allows the study of the precautionary savings motive.

The prudence measure ξ arises from third derivative of the habit utility function being positive in the CC model in Eq. 10. This prudence measure is inversely related to the surplus ratio S_t and is defined by:

$$\begin{aligned} \xi =& - C_t \frac{u_{ccc}(C_t, X_t)}{u_{cc}(C_t, X_t)} \\ =& \frac{C_t(1+\gamma)}{C_t - X_t} = \frac{(1+\gamma)}{S_t} \end{aligned} \tag{17}$$

The unconditional risk-free rate is obtained by taking expectations of Eq. 16 through $r_f = \frac{1}{\mathbb{E}[M_t]}$. The risk-free rate equation with the derivation in the appendix 5.0.1 is:

$$r_{f,t} = \underbrace{-log(\delta) + \gamma g_c + \gamma \alpha_c \epsilon_{x,t-n}}_{intermporal\ substitution} - \underbrace{\gamma(1-\phi_s)(s_t - \bar{s}) - \left[\frac{\gamma^2 \sigma_c^2}{2}(1+\lambda(\cdot))^2\right]}_{precautionary\ savings} \tag{18}$$

The first three terms on the right hand side correspond to the intertemporal rate of substitution while the last two terms pertain to the precautionary savings motive. The third term on the RHS $\gamma\alpha_c\epsilon_{x,t-n}$ comes from the consumption growth process directly and already adjusts the risk-free rate to be non-constant.

The specifications for the equilibrium $\overline{S}$ and $\lambda(\cdot)$ then directly impact the role of the precautionary savings motive in the risk-free rate. In the CC model, the specifications for the $\lambda(\cdot)$ and $\overline{S}$ are in Eqns.5 and 6 which result in a fixed risk-free rate in Eq. 11.

A question arises - does the lagged sentiment further impact the risk-free rate through the precautionary savings motive? To answer this question preliminarily, I first do a OLS regression of the risk free rate against sentiment $\epsilon_{x,t-n}$ and the surplus consumption ratio S_t with the results in table 4. To generate the S_t, I first use the CC model consumption growth Eq. 3 to generate consumption residuals $\epsilon_{c,t}$. This is bias free from any sentiment effect. Starting from an initial S_0 value in 1965Q1 which reflects the economic condition at that period, I then substitute the consumption residuals into Eq. 4 to generate a time series of S_t. In order for more robustness, different initial values of S_0 are used but they are found to not impact the subsequent results much. The natural logarithm of the s_t are used to obtain the regression equation:

$$r_{f,t} = 0.105 + 0.016\epsilon_{x,t-2} + 0.011\epsilon_{x,t-6} + 0.02s_t \tag{19}$$

The detailed results are in Table 4 indicating coefficients that are significant at the 5% confidence level for lags 1 to 6. The scatter plot of the risk free rate with the lagged sentiment

$n = 2$ is in figure 4 which also shows the positive relationship between sentiment shocks with the risk-free rate.[11] The lag $n = 2$ has the highest R^2 of 19.4% highlighting the impact from different lags (one for longer term intertemporal substitution and another for shorter term through the precautionary motive) of sentiment on risk-free rates. The coefficients for $\epsilon_{x,t-n}$ and s_t are all positive showing that they impact the risk-free rate in the same direction, an observation which I used in the subsequent modelling.

[Insert Table 4 and Figure 4 here]

To consider this in the model, I use specifications for the $\lambda(\cdot)$ and the $\overline{S}$ as (with B_1 and B_2 parameters to be estimated):

$$\overline{S} = \sigma_c \sqrt{\frac{\gamma}{1 - \phi - (B_1/\gamma)}} \tag{20}$$

$$\lambda(S_t, \epsilon_{x,t-n}) = \begin{cases} \frac{1}{\overline{S}}\sqrt{1 - 2(s_t - \overline{s}) - B_2\epsilon_{x,t-2}} - 1 & for\ s_t < s_{max} \\ 0 & for\ s_t \geq s_{max} \end{cases} \tag{21}$$

These modified $\lambda(\cdot)$ and $\overline{S}$ result in a log risk free rate $r_{f,t}$ by substitution into Eq. 18 with the derivation in appendix:

$$r_{f,t} = \underbrace{-log(\delta) + \gamma g_c - \frac{\gamma}{2}(1 - \phi_s) + \gamma\alpha_c\epsilon_{x,t-6}}_{intertemporal\ substitution} + \underbrace{\frac{B_1}{2} - B_1(s_t - \overline{s}) + \frac{B_2}{2}[(\gamma(1 - \phi_s) - B_1)]}_{precautionary\ motive}\epsilon_{x,t-2} \tag{22}$$

The specification Eq. 20 means that the equilibrium surplus level $\overline{S}$ is determined parametrically by the B_1 which reflects the impact of the surplus ratio on the risk-free rate. The specification Eq. 21 means that the habit sensitivity is driven by both the s_t and the lagged sentiment shock $\epsilon_{x,t-2}$ which drive the precautionary savings motive as

[11] Note in this case the regression is for the risk free rate whilst later results are for the log risk-free rate which will show opposite sign in the coefficients.

in Eq. 22. Economically, when the surplus consumption ratio s_t increases relative to the $\bar{s}$, prudence decreases according to Eq. 17 and lowers the demand for risk free assets and increases the risk-free rate that is consistent with both the OLS relation Eq. 19 and Eq. 22. When there is a sentiment shock $\epsilon_{x,t-2} > 0$, firms adopt 'less prudence' and increase their investment borrowing. This increases demand and the risk-free rate consistent with the earlier equations as well. This result is confirmed in the empirical GMM section 3.5.

Consider the case for $B_2 = 0$ such that the $\lambda(\cdot)$ sensitivity is *not driven* by the market sentiment, the risk-free rate reduces to:

$$r_{f,t} = -\,log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi) + \gamma\alpha_c\epsilon_{x,t-6} + \frac{B_1}{2} - B_1(s_t - \bar{s}) \quad for\ Model\ 2 \tag{23}$$

For ease of notation, this is denoted by Model 2,[12] which will be tested in the empirical GMM section 3.5. This model adjusts for the equilibrium $\overline{S}$ to include precautionary effect from the surplus ratio although $\overline{S}$ is still a constant that depends on model parameters only. The $\lambda(\cdot)$ depends only on the s_t. In the case of $B_1 = B_2 = 0$, only the intertemporal substitution effect remains driven by the lagged market sentiment $\epsilon_{x,t-6}$. This is denoted as Model 3 with the risk-free rate:

$$r_{f,t} = -\,log(\delta) + \gamma g - \frac{\gamma}{2}(1-\phi) + \gamma\alpha_c\epsilon_{x,t-6} \qquad for\ Model\ 3 \tag{24}$$

In the section 3.5, I do a Generalised Method of Moments and use the Hansen J test over-identifying condition to test the structural validity of these models 1, 2 and 3, and also estimate the parameters B_1 and B_2.

[12]This model is similar to Wachter (2005). However the model in Wachter 05 does not consider market sentiment.

3.4 Equilibrium conditions at steady state

I now consider equilibrium conditions for the Model 1. At $s_t > s_{max}$, Eq. 21 requires the following condition must hold so that the $\lambda(\cdot)$ is well-defined:

$$1 - 2(s_t - \bar{s}) - B_2\epsilon_{x,t-n} \geq \overline{S} \tag{25}$$

Considering the exclusive cases for positive and negative $\epsilon_{x,t-n}$ and solving:

$$B_2 \begin{cases} \geq \frac{(1-2(s_t-\bar{s})-\overline{S})}{\epsilon_{x,t-n}} & for\ \epsilon_{x,t-n} \leq 0 \\ \leq \frac{(1-2(s_t-\bar{s})-\overline{S})}{\epsilon_{x,t-n}} & for\ \epsilon_{x,t-n} \geq 0 \end{cases} \tag{26}$$

Using $s_t = 0.056$ and $\overline{S} = 0.04$, and for the normalised $\epsilon_{x,t-n} = +/-2.0$ (which will cover 99% of the occurrences), $-0.143 < B_2 < 0.143$. The GMM results in table 6 shows B_2 within this range ~ -0.14.

At this boundary s_{max}, the surplus ratio S_t is unresponsive to consumption shocks and setting $\lambda(s_t) = 0$ in Eq. 21 results in:

$$s_{max} = \bar{s} + \frac{1}{2}(1 - \overline{S}^2 - B_2\epsilon_{x,t-n}) \tag{27}$$

In the steady state $\epsilon_{x,t-n} = 0$, $s_{max} \to \bar{s} + \frac{1}{2}(1 - \overline{S}^2)$. This suggests that the lagged sentiment $\epsilon_{x,t-n}$ can change the s_{max} temporally unlike the CC model or Models 2 and 3. A positive $\epsilon_{x,t-n}$ shock with $B_2 < 0$ increases the s_{max}. Economically, with a very high positive sentiment $\epsilon_{x,t-n}$ at the limit, the agent may even *adjust* to have higher surplus expectations s_{max}. In the historical surplus ratios that are generated for 1965Q3 to 2018Q4, this s_{max} is however never reached.

The Model 3 also implies that the consumption C_t is more likely (than the CC model) to fall below the habit level X_t to undesirable effect as it was explained in Ferson and Constantinides (1991), Sundaresan (1989) and Chapman (1998). For this to happen, $S_t <$

0. This is likely to occur with a double whammy of a $\epsilon_{c,t} < 0$ at an already low s_t level and a temporal $\epsilon_{x,t-n} << 0$ shock. This produces a high $\lambda(\cdot)$ such that the S_t momentarily falls to < 0. In the subsequent empirical section between 1965Q3 to 2018Q4, this $S_t < 0$ level has never been breached.

There are two conditions that must be met in the original CC paper in order for the model equilibrium conditions to exist. These are the habit level X_t must be pre-determined both *at* the steady state $s_t = \bar{s}$ itself and *near* the steady state such that it moves non-negatively with consumption everywhere. I verify that these conditions are satisfied for the models.[13] For ease of discussion, I show that the equilibrium conditions are met by restrictions of Model 1 first - Model 2 and 3, and then Model 1 itself.

The equilibrium conditions depend on the specifications of $\lambda(\cdot)$ and the $\overline{S}$. Both the CC model and Model 3 have the same $\lambda(s_t)$ and $\overline{S_t}$ in Eq. 5. In this case, the equilibrium conditions for Model 3 are the same as per the CC model. For the Model 2, the $\lambda(s_t)$ is the same as the CC model and Model 3, but $\overline{S}$ has the expression in Eq. 20. To determine that the habits are stable under steady state conditions, differentiate the transition equation to obtain:

$$\frac{dx_{t+1}}{dc_{t+1}} = 1 - \frac{\lambda_{s_t}}{e^{-s_{t+1}} - 1} \approx 1 - \frac{\lambda_{s_t}}{e^{-s_t} - 1} \tag{28}$$

The latter approximation holds in the steady state as $t \to \infty$. For $\frac{dx_{t+1}}{dc_{t+1}} = 0$ to hold at the steady state $s_t = \bar{s}$ and expanding by Taylor series:

$$\lambda(\bar{s}) = \frac{1}{\overline{S}} - 1 \tag{29}$$

Substituting $s_t = \bar{s}$ in Eq. 5 at the steady state also results in the same Eq. 29 condition, thence ensuring Model 2 and Model 3 both meet this equilibrium condition.

The second condition is for habits to move positively with consumption and such that

[13]There is a third condition of a constant risk free rate that is removed in the models in this paper.

at equilibrium $s_t = \bar{s}$, $\frac{d}{ds}(\frac{dx}{dc})|_{s=\bar{s}}= 0$. This condition restricts the $\frac{dx}{dc}$ to be a U-shaped graph with respect to s_t. Differentiating the Eq. 28 respect to s_t, setting it to zero at $s = \bar{s}$ results in Eq. 30 which is satisfied by the $\lambda(s_t)$ specifications for Models 1 and 2 for $s_t = \bar{s}$:

$$\lambda'(\bar{s}) = -\frac{1}{\overline{S}} \tag{30}$$

The preceding arguments show that the equilibrium conditions are similar for Models 1 and 2 as the CC model. The addition of a fixed parameter $\frac{B_1}{\gamma}$ in Eq. 20 for $\overline{S}$ does not alter the equilibrium conditions for Model 2. The specification of the lagged sentiment shock (which is stationary) in the consumption growth process also does not alter equilibrium.

For the Model 3, the $\lambda(\cdot)$ specification in Eq. 21 further requires $\epsilon_{x,t-n} \to 0$ at steady state. By virtue of the auto-regressive Eq. 2 for the sentiment this is satisfied by the unconditional expectation $E(\epsilon_{x,t-n}) = 0$. This result means that in the absence of any further sentiment shock, the Model 3 equilibrium conditions revert to the same as Model 2, whenceforth the solution for $\lambda(\cdot)$ in Eq. 30 is the same obtained from differentiation of $\frac{dx_{t+1}}{dc_{t+1}} = 0$.

3.5 Generalised Method of Moments

In order to verify my postulation of the better empirical fit of Model 3, I formulate a series of moments conditions tested using GMM. I run GMM on Model 1 and its subset restrictions - Model 3 with ($B_1 = B_2 = 0$), Model 2 with $B_2 = 0$ with 3 moment equations indicated below in m_1, m_2 and m_3. The first moment is for the expected risk-free rate with equivalent regression results in table 4. The second moment is for the consumption growth with corresponding regression results in table 3 using the $\epsilon_{x,t-6}$ as the dependent variable since lag $n = 6$ shows the best R^2 result. The third moment is for the long run variance of annual consumption growth estimated from 1965Q3 to 2018Q4 at 0.98% and is used to account for the known smooth nature of consumption growth.[14].

- m_1: Expected risk-free rates
- m_2 : Expected consumption growth rates
- m_3 : Long run variance of consumption growth σ_c

For Model 3, the moment conditions are:

$$\mathbb{E}\begin{bmatrix} m_1 = r_{f,t} - \{-log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi) + \gamma\alpha_c\epsilon_{x,t-6} + \frac{B_1}{2} - B_1(s_t - \bar{s}) + \frac{B_2}{2}[\gamma(1-\phi_s) - B_1]\epsilon_{x,t-2}\} \\ m_2 = \Delta c_{t+1} - \{g_c + \gamma\alpha_c\epsilon_{x,t-6}\} \\ m_3 = m_2 * m_2 - \sigma_c^2 \end{bmatrix} = 0 \tag{31}$$

The instrumental variables used include the lagged sentiment shocks $\epsilon_{x,t-2}$, $\epsilon_{x,t-6}$ and a constant. Since the number of moments is greater than the number of parameters to be estimated, the over-identifying conditions are reported with the Hansen J'stat using the HAC weighting matrix updated in a 2-step GMM methodology in Hansen (1982). Note in the original CC model, the log risk-free rate is restricted at 0.94 from his paper which will fail the m_1 moment. The significance of the $\alpha_c \neq 0$, $B_1 \neq 0$ and $B_2 \neq 0$ in the subsequent results further justify the Models 1, 2 and 3 relative to the CC model.

In order to first generate the s_t for the Model 3, I have to presume the value of $B_1$and B_2 values since the historical s_t are generated from Eq. 21 and Eq. 13. This is done through a trial and error approach until the presumed and the estimated match within 0.005. The convergence is relatively stable. The results show the estimated parameters $\alpha_c = -0.0026$, $B_1 = 0.0096$ and $B_2 = -0.14$. In row no 4, the parameter ϕ_s is a free parameter to be optimised in Model 3. The results show it to be invariant (0.869 to 0.87) from the row no 3 of Model 3, indicating that the habit persistence has not changed in spite of adding the

[14] The construction of the consumption growth rates follow the example in Schorfheide et al. (2018) with the data in link.

lagged sentiment as a driver of the habit sensitivity.

The moment conditions for Model 2 differ only by the first moment condition for the risk-free rate as:

$$\mathbb{E}\left[m_1 = r_f - \{-log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi) + \gamma\alpha_c\epsilon_{x,t-6} + \frac{B_1}{2} - B_1(s_t - \bar{s})\}\right] = 0 \tag{32}$$

In order to first generate the s_t for the test, I have to similarly presume the value of B_1 as for Model 3. The results show the estimated parameters $\alpha_c = -0.0035$ and $B_1 = 0.012$.[15] The results are in row 2.

For Model 3, the moment conditions differ only by m_1 with only the $\alpha_c = -0.0032$ estimated:

$$m_1 = r_f - \{-log(\delta) + \gamma g - \frac{\gamma}{2}(1-\phi) + \gamma\alpha_c\epsilon_{x,t-2}\} \tag{33}$$

The Hansen J stat for the 3 models all reject the null hypothesis of structural invalidity with p-values of 0.23, 0.28 and 0.20 for Model 1, 2 and 3 respectively. The results imply that Model 2 is most probable with the lagged sentiment impacting intertemporal substitution and the surplus ratio impacting the precautionary savings motive. The Model 1 is the next probable with the lagged sentiment impacting the precautionary savings motive in addition.

[Insert Table 6 here.]

With the GMM results, I now analyse Eq. 22 to weigh the effects of the intertemporal substitution and the precautionary motives by sentiment on Model 1 risk-free rate. Substitution of the calibrated parameter values for the coefficient of $\epsilon_{x,t-n}$ returns $\gamma\alpha_x\epsilon_{x,t-n} = -0.006$ and $\frac{B_2}{2}(\gamma(1-\phi_s) - B_1) = -0.0175$ showing the sentiment precautionary measure to have 74% weightage. The coefficient of the $(s_t - \bar{s})$ is -0.009 that also contributes to the precautionary motive. The $-log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi_s)$ intertemporal substitution term is

[15] As a comparison, this estimated $B_1 = 0.011$ in Wachter (2005).

-0.032. There is an additional dead-weight constant of $\frac{B_1}{2} = 0.0048$ in the model that contributes to precautionary motive. The actual contribution is dependent on the realisations of the $\epsilon_{x,t-n}$ and the s_t term, although both act in the same direction consistent with the OLS regression in Eq. 19.

The impact of the firm sentiment on risk-free rates needs to be considered from both the supply (firm) and the demand (side) consumers. Suppose the realisations for $\epsilon_{x,t-n} = 0.5$ for both lags $n = 2$ and 6, and $S_t - \overline{S} = 0.01$ which is 'normal economic times', the risk free rate will be 5.32% in Model 1. This contrasts with the CC model of a fixed 4.4%. Model 2 returns 4.43% and Model 3 of 4.72%. In this case, the Model 1 and Model 3 provides a higher risk-free rate than the CC model due to demand from the positive sentiment of *firms* which want to borrow more causing rates to increase. In addition, Model 1 has an indirect impact through the *consumer* precautionary motive from the lagged sentiment *increasing* the habit sensitivity of consumers. This increases the surplus ratio and reduces the prudence, reducing savings from the supply side, which in turn causes the risk-free rate to increase to stimulate savings. This explains the higher rate of the Model 1. On the contrary, the Model 2 risk-free rate decreases as the additional modelling of prudence relative to the CC increases savings.

Suppose the realisations for $\epsilon_{x,t-n} = -0.5$ and $s_t - \bar{s} = 0.01$ which is 'recessionary economic times', the risk free rate will be 2.8% in Model 1. In this scenario, the negative sentiment makes the firms more prudent and borrow less, decreasing the risk-free rate relative to the fixed CC rate. Comparatively the Model 2 rate is 3.53% and the Model 1 rate is 4.1%. In recent times, recessionary periods tend to associate with especially low interest rates.

I now consider the impact on the equilibrium surplus level $\overline{S}$ which is sensitive to B_1 according to Eq. 20. The value of $B_1 = 0.0096$ in Model 3 is lower than in Model 2 with $B_1 = 0.012$. This causes a marginal 1.0% decrease (or increase) in $\overline{S}$ (or $\overline{X}$) level when the lagged sentiment shock is factored into Model 3 $\lambda(\cdot)$. Since the B_1 is obtained from calibration, it may be attributed to the positive skew of the Baker-Wurgler sentiment in table 1. Economically speaking, with a positive sentiment shock from the markets, consumers raise their habit expectations for consumption.

An analysis on the impact of increasing risk coefficient γ shows an increase (or decrease) of the equilibrium $\overline{S}$ (or $\overline{X}$) level in figure 5. This result is common across all models - Models 1, 2 and 3 and even the CC Model. In Guiso et al. (2018), the risk aversion coefficient had been found to increase post the Lehman crisis for Italian households. The economic intuition is that agents who are more risk averse may lower their habit expectations in this post-recession period.

[Insert Figure 5 here]

3.6 Stochastic Discount Factor

I next turn my analysis to risky assets. Following standard derviation, the Euler's equation for consumption is:

$$\frac{P_t}{C_t} = \mathbb{E}_t\left[M_{t+1}\left((\frac{P_{t+1}}{C_{t+1}}+1)\frac{C_{t+1}}{C_t}\right)\right] \tag{34}$$

The analysis below applies to all Model 1 and its restrictions Models 2 and 3 since these models have the same consumption growth equation and differ only by the functional form of the $\lambda(\cdot)$ and $\overline{S}$. Let $G(\epsilon_{x,t-n}, s_t)$ denote the solution for the price consumption ratio as functions of the lagged sentiment shock $\epsilon_{x,t-n}$ and the surplus ratio s_t. The Eqns. 12 and 16 for the consumption growth and the stochastic discount factor respectively are substituted into Eq. 34 to result in:

$$\begin{aligned} G(\epsilon_{x,t-n}, s_t) =& \mathbb{E}_t\left[\delta e^{-\gamma[g_c+\alpha_c\epsilon_{x,t-n}+(1-\phi_s)(\overline{s}-s_t)+(1+\lambda(\cdot))\epsilon_{c,t}]}e^{(g_c+\alpha_c\epsilon_{x,t-n}+\epsilon_{c,t})}(G(\epsilon_{x,t-n}, s_{t+1})+1)\right] \\ =& \mathbb{E}_t\left[\delta e^{(1-\gamma)(g_c+\alpha_c\epsilon_{x,t-n})-\gamma(1-\phi_s)(\overline{s}-s_t)+[1-\gamma(1+\lambda(\cdot)]\epsilon_{c,t}}(G(\epsilon_{x,t-n}, s_{t+1})+1)\right] \end{aligned} \tag{35}$$

The functional $G(\epsilon_{x,t-n}, s_t)$ depends unconditionally on the $p(\nu_{x,t-n}, \nu_{c,t})$ the bivariate probability density of $\epsilon_{x,t-n}$ and $\epsilon_{c,t}$ as below.

$$p(\nu_{x,t-n}, \nu_{c,t}) = \frac{1}{2\pi\sigma_x\sigma_c\sqrt{1-\rho_{x,c}^2}} exp\big(-(\frac{z}{2(1-\rho_{x,c}^2)}\big) \tag{36}$$

$$where \quad z = \frac{\nu_{x,t}^2}{\sigma_{x,t-n}^2} + \frac{\nu_{c,t}^2}{\sigma_c^2} - \frac{2\rho_{x,c}\nu_{x,t-n}, \nu_{c,t}}{\sigma_x\sigma_c} \tag{37}$$

Substituting this into equation 35 gives:

$$\begin{aligned} G(\epsilon_{x,t-n}, s_t) = \delta e^{g_c(1-\gamma)-\gamma(1-\phi_s)(\bar{s}-s_t)} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(\nu_{x-n,t}, \nu_{c,t}) e^{\alpha_c(1-\gamma)\nu_{x-n,t}+[1-\gamma(1+\lambda(\cdot)]\nu_{c,t}} \\ (G(\epsilon_{x,t-n}, s_{t+1})+1)d\nu_{x-n,t}d\nu_{c,t} \end{aligned} \tag{38}$$

I solve this equation as conditional on the lagged sentiment shock $\epsilon_{x,t-n}$. There are two justifications for this. Firstly, the sentiment evolves as AR(1) with a relatively persistent coefficient (~ 0.95) in Eq. 2. Secondly and trivially, for sentiment shock $\epsilon_{x,t-n} = \epsilon_{\tilde{x},t}$ is known at time t and its density function is given by:

$$\nu_{c,t}|_{\epsilon_{x,t-n}=\tilde{x}_t} \sim \mathcal{N}(\frac{\sigma_c}{\sigma_x}\rho_{x,c}\epsilon_{\tilde{x},t}, (1-\rho^2)\sigma_c^2) \tag{39}$$

The recursive Euler's equation to solve is now:

$$\begin{aligned} G(\epsilon_{\tilde{x},t}, s_t) = \delta e^{g_c(1-\gamma)-\gamma(1-\phi_s)(\bar{s}-s_t)} \int_{-\infty}^{\infty} p(\nu_{c,t}|x_t = \epsilon_{\tilde{x},t})) e^{\alpha_c\tilde{x}(1-\gamma)+[1-\gamma(1+\lambda(\cdot)]\nu_{c,t}} \\ [G(\epsilon_{\tilde{x},t}, s_{t+1})+1]d(\nu_{c,t}|\epsilon_{x,t-n} = \epsilon_{\tilde{x},t}) \end{aligned} \tag{40}$$

This Eq. 40 can be solved either as a fixed point problem in the original CC paper, or a series of zero coupon equity cash flows in the Wachter (2005). Both require an initial double grid values for $\epsilon_{c,t}$ and the sentiment shock $\epsilon_{x,t-n}$. In this conditional distribution case, the sentiment shock has effectively shifted the mean of the consumption growth shock from 0 to $\frac{\sigma_c}{\sigma_x}\rho_{x,c}\epsilon_{\tilde{x},t}$ and decreased its variance by $\rho_{x,c}^2$. Since the long-run correlation

$\rho_{x,c} \sim 0.09$ in table 2, this shift in the mean and variance is relatively small, although the $\rho_{x,c}$ is notoriously difficult to estimate at points in time. Eq. 40 is solved for different empirical values of the sentiment $\epsilon_{x,t-n}$ from -2.0 to 2.0 which act like 'slices' on the 3-dimensional surface. The parameters for the different model solutions are listed in table 5.

[Insert Table 5 here]

The price-consumption surface in Eq. 40 is solved by a Python program available in the link. It uses the scipy function quad for integration. The figure 6 shows the price-consumption surfaces solving the fixed point solution for Eq. 40 for the models - CC in figure 6a, Model 3 in figure 6d, Model 2 in figure 6c and Model 1 in figure 6b.

[Insert Figure 6 here]

There are two points to glean from the price-consumption surface. These are the steepness relative to the surplus ratio and the relative tilt of the positive sentiment to negative sentiment. The former point relates to the volatility of the stochastic factor M_t relative to the surplus ratio. The latter point relates to the cyclical properties of sentiment in the discounting premium. The downward tilt for Model 1 towards negative sentiment is the most amongst the models, which makes intuition since it considers in addition the habits sensitivity to sentiment. The argument that risk-taking is greater with positive sentiment is also consistent with the higher price-consumption ratios observed in the tilt. A greater risk-taking implies a greater risk premium, which from Eq. 46 would imply a greater tilt and higher PC_{t+1} values for the positive sentiment. To understand this, suppose the sentiment improves from negative to positive increasing the $\lambda(\cdot)$ habit sensitivity. The agent becomes more sensitive to a same consumption shock which increases the s_{t+1}, which decreases the M_{t+1} and increases the risk premium.

Comparatively, an increase in the surplus ratio S_t from 0.03 to 0.05 (for zero sentiment shock) increases the price consumption ratio from 16.25 to 29.2 (estimated returns of 185%), whilst the increase in the sentiment from -1.5 to 1.5 (which are the observed limits of the sentiment and when the surplus ratio is near equilibrium at 0.04) is from 20.0 to 24.34 (estimated returns of 24%) respectively. This shows that the surplus ratio is still the main driver of risk premium compared to market sentiment although the importance of the sentiment cannot be ignored. This leads to the next section 3.7 on the market price of risk for the 3 models.

3.7 Market Price of Risk

This section discusses the market price of risk for Models 1, 2 and 3. The market price of risk is defined as the ratio of the first and second moment of the risky asset return. The Hansen and Jaganthan bound in Hansen and Jagannathan (1991) is an upper bound on the Sharpe ratio of an equity return R^e_{t+1} when $\rho_t(M_{t+1}, R^e_{t+1}) = -1$:

$$\frac{\mathbb{E}_t[R^e_{t+1}]}{\sigma(R^e_{t+1})} = -\rho_t(M_{t+1}, R^e_{t+1})\frac{\sigma(M_{t+1})}{\mathbb{E}_t[M_{t+1}]} \leq \frac{\sigma(M_{t+1})}{\mathbb{E}_t[M_{t+1}]} \tag{41}$$

The base stochastic discount factor for all 3 models is expressed in Eq. 16. The stochastic discount factor $M_t = e^{\mu_m + \sigma_m \epsilon_c}$ where μ_M and σ_M are its lognormal mean and standard deviation respectively. The maximum Sharpe ratio for all models is derived in the appendix 5.0.2 and depends on the σ_M *only*:[16]

$$max\ sharpe\ ratio = \sqrt{e^{\sigma_m^2} - 1} \tag{42}$$

This σ_M is derived in the appendix Eq. 54 for the restricted Models 2 and 3:

$$\sigma_M = (1 + \lambda(s_t))\sigma_c \tag{43}$$

$$= [\frac{1}{\overline{S}}\sqrt{1 - 2(s_t - \overline{s})}]\sigma_c \tag{44}$$

[16]This is a same result in the original CC paper.

In the case of Model 3, it has the same Sharpe ratio as the original CC model (see summary table 8. The sentiment risk $\epsilon_{x,t-n}$ is not priced if the lagged sentiment state variable appears only in the drift component. For Model 2, the $\overline{S}$ has an additional term $\frac{B_1}{\gamma}$ in Eq. 20 which increases the Sharpe ratio since $B_1 > 0$. This increase is however independent of the state variables, since $\frac{B_1}{\gamma}$ is a fixed constant. For Model 1, there is an additional term $-B_2\epsilon_{x,t-n}$ in the Sharpe ratio:

$$\sigma_M = \left[\frac{1}{\overline{\overline{S}}}\sqrt{1 - 2(s_t - \overline{s}) - B_2\epsilon_{x,t-n}}\right]\sigma_c \tag{45}$$

This additional term decreases the Sharpe ratio further (for $\epsilon_{x,t-n} < 0$ since $B_2 < 0$) and is also stochastically dependent on the $\epsilon_{x,t-n}$ term. This is a counter-intuitive result since the Sharpe ratios are usually higher in recession and negative sentiment scenarios. However, unlike the surplus ratio which is pro-cyclical, habits formation are counter-cyclical,[17] and the lagged sentiment shock works through this habit sensitivity. In a recession, negative sentiment decreases the habit sensitivity which is contrary to a low surplus ratio increasing the sensitivity. This decrease in the habit sensitivity in turn reduces the Sharpe ratio.

Since the volatility movements of $2(s_t - \overline{s}) > B_2\epsilon_{x,t-n}$, the impact of the surplus ratio s_t is still more important in determining Sharpe ratios, a result which is verified by the earlier discussion on the surplus ratio being a more important driver of the risk premium than the sentiment. Further note that the $\mathbb{E}[\epsilon_{x,t-n}] = 0$, which evens outs the sentiment impact, unlike the surplus ratio which averages about $0.01 - 0.02$ in normal economic times observed in the data.

[17] Habits have an inverse relation with the surplus ratio, as pointed out earlier.

4 Discussion of Model Economic Implications

I next discuss the model economic implications as regards to its historical performance and a partial behavioural explanation of the value premium.

4.1 Historical Performance of Consumption CAPM Models

I next compare the performances of Models 1, 2 and 3 in modelling the risky rates regards to the value weighted quarterly returns from CRSP. To do so, I use the historical residuals of the Baker-Wurgler sentiment index and generate the surplus ratios from Eq. 13 that were earlier used in the GMM moment equations. These are then used to compute equity returns in the standard way as:

$$R_{t+1} = \frac{[PC_{t+1}(s_{t+1}, \epsilon_{t-n+1}) + 1]}{PC_t(s_t, \epsilon_{t-n})} \frac{C_{t+1}}{C_t} \tag{46}$$

To obtain the price-consumption ratios from the models, the surface figures 6d, 6c and 6b from Models 1, 2 and 3 are used respectively in each quarterly step interpolation for $PC_t(s_t, \epsilon_{t-n})$. In this manner, a time series of the model simulated returns are obtained from 1965Q3 to 2018Q4. An independent regression of the historically realised CRSP value weighted returns is then done against the simulated returns and results are in table 7.

[Insert Table 7 here]

The simulated returns for the 3 Models are very close with the Model 1, 2 and 3 having R^2 of $16.7\%, 16.8\%$ *and* 16.6% in fitting the historical data. There is need for caution for the grid granularity used in the fixed point solution for the PC surface (See Wachter (2005)) which can affect the regression results. The PC ratios generated for the returns are generally quite close to one another, with the same historical consumption growth $\frac{C_{t+1}}{C_t}$ used in all models. The results compare well to Kelly et al. (2018b) where the use of non-linear machine learning models and greater number of factors was able to push the in-sample R^2 to just above 21%.

In the popular book 'Irrational exuberance' by Shiller (2013), Shiller reported that financial crises are caused not only by fundamental shocks but also the *emotions of people*. This was also described as *animal spirits* by Keynes to describe the instincts and emotions that dictate human behaviours that influence consumer spending and confidence.

Since the Model 1 is impacted by both market sentiment and the fundamental surplus ratio, I analyse their impacts on the markets. To do so, I generated the historical price-consumption ratios as shown in the historical chart 7 along with the surplus ratio, the market sentiment and the PC ratios (from both Model 1 and CC model). The figure shaded areas relate to the periods of financial crisis - the oil crisis in 1973, the early 1980s caused by the Federal Reserve stagflation policy, the Black Monday in 1987 that eventually impacted the main economy or consumption, the dot-com crisis in early 2000 and the Lehman crisis in 2008. All these crisis correlated with low surplus ratios and the fundamental economy. The PC ratios generated by the two models are close in values and the market sentiment runs pro-cyclically with the fundamental economy generally. These points and the discussion in the market price of risks suggests that the surplus ratio is still the main driver of risk in the economy.

[Insert Figure 7 here]

4.2 Sentiment, Habits and the Value Premium

I now associate the agent's behaviour under the habit utility model to the value premium in the macro-economy. Consider again the same economy with a risky asset and a risk-free asset. At time $t = 0$, the agent observes a positive earnings stream for the risky asset and reacts conservatively to it in the prevailing negative market sentiment. This agent's conservatism lowers his price projection and through the $-\gamma\alpha_c\epsilon_{x,t-n} < 0$ the stochastic discount factor M_{t+1} in Eq. 16 is also decreased. At time $t = 1$, investors correct their initial conservatism and the sentiment mean reverts bringing prices back up to equilibrium and generating a positive premium. Since value stocks are associated with greater earnings stream than growth stocks, they are more affected by this phenomenon effect.[18]

The value premium may also be understood pictorially. The idea is this - starting from a point on the price-consumption surface in figures 6b, 6c and 6d for Models 1 to 3 on an initial low surplus ratio and negative sentiment value, a simulation 'moves' the point about on the surface. If the 'point' moves to a higher surface with greater price-consumption ratios in a pull to equilibrium of the sentiment auto-regressive equation, higher returns and a premium are generated.[19] Historically, observing the table 1, the majority of the sentiment shocks are negative but when they recover, the positive shocks are very much skewed to the right. On the contrary, the price-consumption ratios are invariant to sentiment in the CC model and not generate a value premium through this behavioural aspect.

5 Conclusion

The determination of risk is a key concern in asset pricing. In this paper, two important state variables are used to account for this risk - the fundamental surplus ratio and the market sentiment shocks. The paper shows that positive sentiment increases habit sensitivity and habit levels at equilibrium are marginally increased. These higher habit levels and increased sensitivity to consumption shocks are what makes agents to be greater risk-takers in a positive sentiment environment. This increased risk-taking (lack of prudence) further prompts agents to invest less in risk-free assets and increases the risk-free rate. This positive relation is observed empirically in both GMM and OLS regression results.

On its own, the intertemporal substitution effect of market sentiment on consumption is not priced as a risk factor, but only when it is included into the habit sensitivity, it has a

[18] Wang (2018) has the similar explanation to the value premium although the paper was written from an accounting perspective.

[19] The market sentiment moves to an unconditional mean of $\frac{c_{bw}}{1-\phi_{bw}} \approx 0.01$ although the c_{bw} is estimated with a low degree of precision.

market price of risk. The market sentiment shocks impact the stochastic drifts of both the consumption and dividend growths, and creates a 'tilt down' of the price-dividend and price-consumption surfaces towards negative sentiment. This 'tilt up' towards positive sentiment further supports that agents are risk-takers in a positive sentiment environment since increased risk-taking means a greater risk premium amid higher price-consumption ratios. I use this price consumption ratio surface tilt to explain the value premium with and without the sentiment effect (against the CC Model) and use the behavioural basis (conservatism) cited in Barberis et al on how agent behave in a market-wide pessimism.

I now turn to the two questions posed at the start of this paper. *Does market sentiment reinforce or diminish consumption habits?* A positive market sentiment shock increases habit sensitivity and raises equilibrium habit levels marginally, but does not change habit persistence. *Does positive market sentiment make agents more or less prudent to save or even borrow to maintain their consumption habits?* No, it makes agents less prudent to save, and more inclined to borrow to maintain consumption habits.

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Description	Value	Probability
Skewness	0.54	
Kurtosis	3.25	
Jarque-Bera	5.45	0.065
AR(1) model		
ϕ_x	0.95	0.00
c_{bw}	0.15	0.73
σ_x	0.275	

The t-stats are White Heteroscedasticity adjusted.

$$x_{x,t+1} = \phi_x x_{x,t} + c_{bw} \tag{47}$$

The Baker-Wurgler sentiment residuals are more volatile than the consumption growth - 27.5% annualised. They also show a relatively positive skewness which reflects times of exuberance in the economy.

Table 1: Statistical properties of the Baker-Wurgler index

Parameter	$\epsilon_{d,t}$	$\epsilon_{c,t}$	$\epsilon_{x,t}$
$\epsilon_{d,t}$	1.0	0.21	-0.074
$\epsilon_{c,t}$		1.0	0.09
$\epsilon_{x,t}$			1.0

These are quarterly long-run correlations between the consumption growth, sentiment shocks and the dividend growth between 1965Q3 to 2018Q4.

Table 2: Correlation Matrix of sentiment, dividend and consumption growth shocks

Consumption growth				Dividend growth			
	Coefficients				Coefficients		
No of lags	α_c	g_c	R^2	No of lags	α_d	g_d	R^2
1	0.0017	0.004	0.01	1	-0.004	0.013	0.002
	(0.15)	(0.00)			(0.85)	(0.03)	
2	0.0015	0.004	0.004	2	-0.037	0.013	0.014
	(0.33)	(0.00)			(0.01)	(0.03)	
3	-0.0008	0.004	0.002	3	-0.011	0.013	0.001
	(0.46)	(0.00)			(0.60)	(0.03)	
4	-0.0022	0.004	0.016	4	-0.039	0.014	0.016
	(0.07)	(0.00)			(0.067)	(0.02)	
5	-0.0011	0.004	0.003	5	-0.017	0.0137	0.002
	(0.38)	(0.00)			(0.44)	(0.01)	
6	**-0.0027**	0.004	0.023	6	**-0.049**	0.013	0.025
	(0.02*)	(0.00)			(0.02*)	(0.03)	
7	-0.0014	0.004	0.006	7	-0.011	0.013	0.001
	(0.24)	(0.00)			(0.61)	(0.03)	
8	-0.001	0.004	0.003	8	-0.005	0.013	0.002
	(0.41)	(0.00)			(0.83)	(0.035)	

The dependent variable is the quarterly consumption growth and tested against various order of lags of the sentiment . The p-values are in brackets and are White heteroscedasticity adjusted. The * indicates significance at the 5% confidence level. The data is from 1965Q3 to 2018Q4 for a total of 207 data points for the full period. The R^2 results show that the order of lag n = 6 quarters is the most likely for which the lagged market sentiment drives consumption and dividend growths.

$$\begin{aligned} ln(C_{t+1}/C_t) &= g_c + \alpha_c \epsilon_{x,t-n} + \epsilon_{c,t} \quad \epsilon_{c,t} \sim N(0, \sigma_c) \\ ln(D_{t+1}/D_t) &= g_d + \alpha_d \epsilon_{x,t-n} + \epsilon_{d,t} \quad \epsilon_{d,t} \sim N(0, \sigma_d) \end{aligned}$$

Table 3: Regression results for dividend and consumption growths against lagged sentiment shocks

No of lags n	Lagged sentiment $\epsilon_{x,t-n}$	Lagged sentiment $\epsilon_{x,t-6}$	Surplus ratio s_t	Constant	R^2
1	0.015	0.013	0.02	0.198	0.188
	(0.02*)	(0.03*)	(0.00)	(0.00)	
2	0.0163	0.011	0.0197	0.105	0.194
	(0.011*)	(0.07)	(0.00)	(0.00)	
3	0.014	0.0086	0.0198	0.106	0.186
	(0.03*)	(0.176)	(0.00)	(0.00)	
4	0.009	(0.007)	0.0199	0.106	0.173
	(0.19)	(0.272)	(0.00)	(0.00)	
5	0.012	-	0.02	0.106	0.165
	(0.05*)	-	(0.00)	(0.00)	
6	0.011	-	0.02	0.107	0.161
	(0.067)	-	(0.00)	(0.00)	

The dependent variable is the risk free rate and tested against various order of lags n of the sentiment and the surplus ratio generated from the original CC model. The p-values are in brackets below and White heteroscedascity adjusted. The R^2 results show that the lagged sentiment has causative impact on the risk-free rate likeliest at lag $n = 2$. However, the coefficients of lags from 1 to 3 are significant at the 5% level. The results show that both the surplus ratio and the lagged sentiment shocks impact the risk-free rate in the same direction. From row 3 onwards, only a single lag is used for the regression due to multi-collinearity between the lags. The * indicates significance at the 5% level.

$$r_{f,t} = c + \beta_1 \epsilon_{x,t-n} + \beta_2 \epsilon_{x,t-6} + \beta_s s_t \tag{48}$$

Table 4: Impact of lagged sentiment and surplus ratio on the risk free rate

Description	Notation	Value				
Common Parameters						
Utility curvature	γ	2				
Subjective discount rate	δ	0.925				
Model Parameter Values						
		CC (original model)	CC (newly calibrated)	Model 3	Model 2	Model 1
Consumption growth rate constant	g_c	0.0189	0.0164	0.0161	0.0161	0.0161
Consumption growth volatility	σ_c	0.015	0.01	0.0098	0.0098	0.0098
Habit persistence	ϕ_s	0.87	0.87	0.87	0.87	0.87
Equilibrium surplus ratio	$\overline{S}$	0.057	0.0392	0.034	0.042	0.040
Maximum surplus ratio	S_{max}	0.094	0.065	0.063	0.070	$f(\epsilon_{x,t-n})$

The parameters are all annualised and estimated using quarterly data from 1967Q2 to 2018Q4 with 207 data points. The estimates in the original CC paper are annualised from 1889 to 1992 and differ from table as it has a higher consumption volatility σ_c. The subjective discount factor δ used in the original CC paper is 0.89 whilst 0.925 is used here to reflect the lower interest rate environment since 2000s.

Table 5: Model constants

No	Model	ϕ_s	Parameter est. α_c	B_1	B_2	Instrumental Variables Eq1	Eq2	Eq3	Hansen's stat	p-value
1	Model 1	0.87*	-0.0026 (0.025)	0.0096 (0.00)	-0.14 (0.005)	$\epsilon_{x,t-2}$,c	$\epsilon_{x,t-6}$	$\epsilon_{x,t-6}$	1.51	0.22
2	Model 2	0.87*	-0.0035 (0.01)	0.012 (0.00)	-	$\epsilon_{x,t-6}$, c	$\epsilon_{x,t-6}$	$\epsilon_{x,t-6}$	2.47	0.28
3	Model 3	0.87*	-0.0032 (0.00)	-	-	$\epsilon_{x,t-6}$, c	$\epsilon_{x,t-6}$	$\epsilon_{x,t-6}$	4.6	0.20
4	Model 1	0.869 (0.00)	-0.0025 (0.033)	0.01*	-0.14 (0.004)	$\epsilon_{x,t-2}$,c	$\epsilon_{x,t-6}$	$\epsilon_{x,t-6}$	1.46	0.226

The parameters are all annualised and estimated on quarterly data from 1965Q3 to 2018Q4 for 207 data points. Comparing rows 1 to 3, the Model 2 has the highest Q(b) criterion from the GMM estimation and is the most probable model. All Models 1 to 4 are not rejected at the 5% significance level. In the rows 1 and 4 for Model 3, the parameters that are fixed to be a constant are denoted by an asterisk - ϕ_s in rows 1-3 and B_1 in row 4. The row 4 is to determine the habit persistence ϕ_s when optimised as a free parameter in the model. Both rows return the same result of $\phi_s \approx 0.87$, indicating that the inclusion of the lagged sentiment as an exogenous risk factor through B_2 into the habit sensitivity function $\lambda(\cdot)$ does not impact the ϕ_s value much.

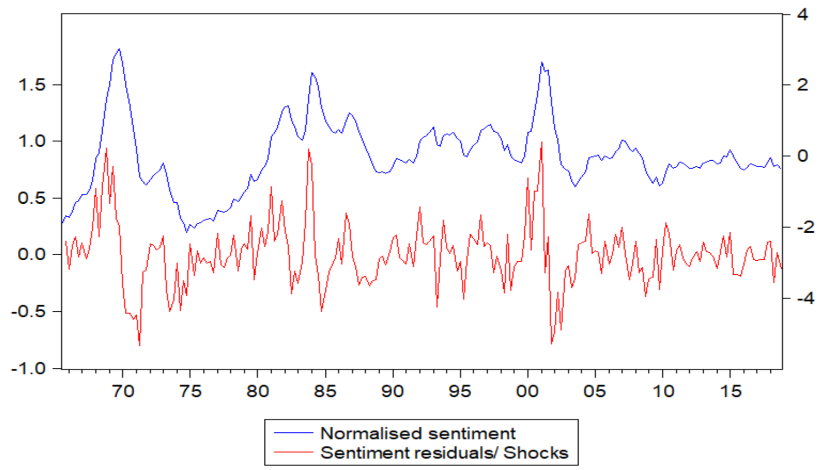
Table 6: GMM Results for all Models

	Models		
	1	2	3
ϕ_s	0.87	0.87	0.87*
Coefficient	1.89	1.71	1.88
	(0.00)	(0.00)	(0.00)
R^2	0.173	0.155	0.172
Log-likelihood	437.4	435	437.3

The table shows the regression results of the CRSP value-weighted returns against returns generated independently by the price-consumption function plot from the models in figures 6. The Model 1 shows the highest R^2 of 17.3%, an improvement of 1.8% relative to the Model 2, when sentiment is added to the habit sensitivity. Notice in this case, with the higher R^2 the coefficient size has also got larger in its ability to explain the CRSP returns. The improvement in the Model 3 comes primarily from the modelling of the stochastic discount factor, and not from projected cash flows which would have come from a price-dividend plot and dividend growth. Comparatively, Kelly et al. (2018b) uses more factors and non-linear machine learning methodology to achieve the highest R^2 of about 21%.

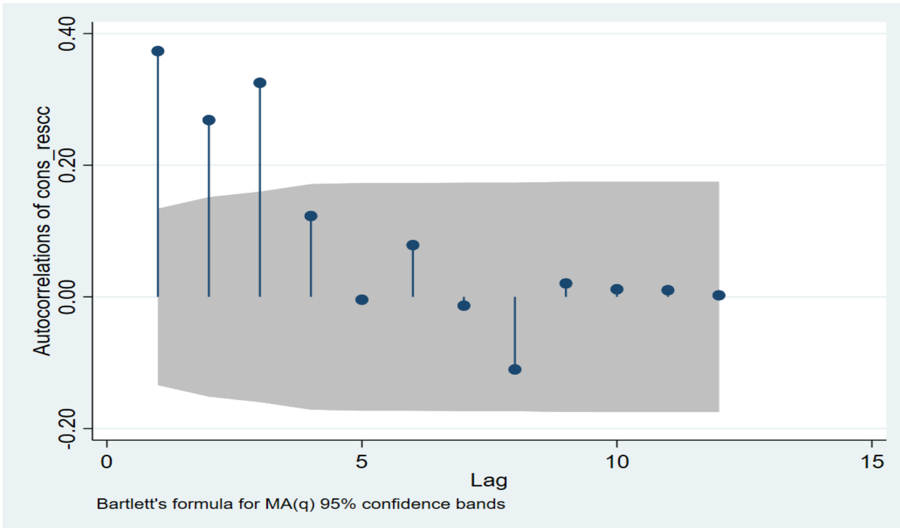
Table 7: Historical Simulation Returns Regression against the CRSP VWRET

Figure 1: Time Series of Baker-Wurgler sentiment (normalised) and AR(1) sentiment shocks



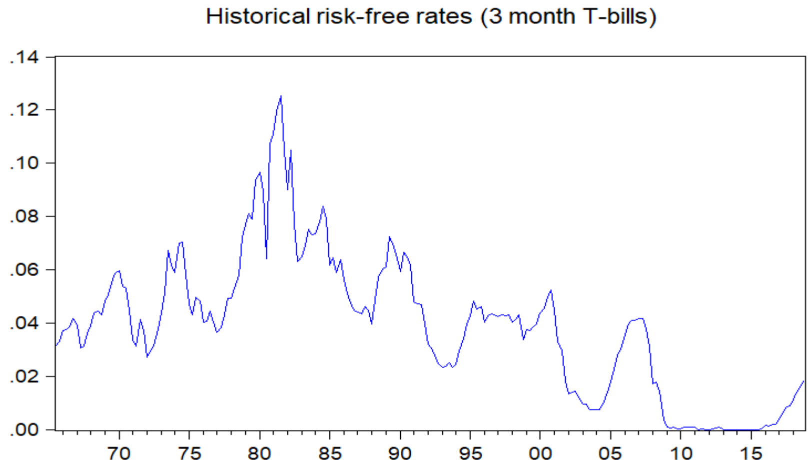
The figure shows a time series of the Baker-Wurgler (normalised) sentiment Baker and Wurgler (2007) and its residuals from an AR(1) model.

Figure 2: Correlogram of the consumption growth



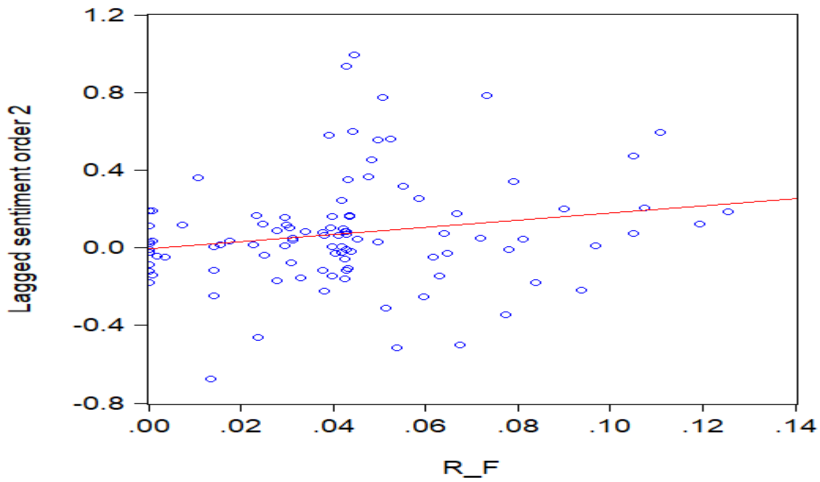
The figure shows the acf correlogram of the consumption growth. At lags 1 to 3, the acf is significantly positive, but for greater lags, it turns zero or even negative.

Figure 3: Plot of risk free rate from 1965Q4 to 2018Q4



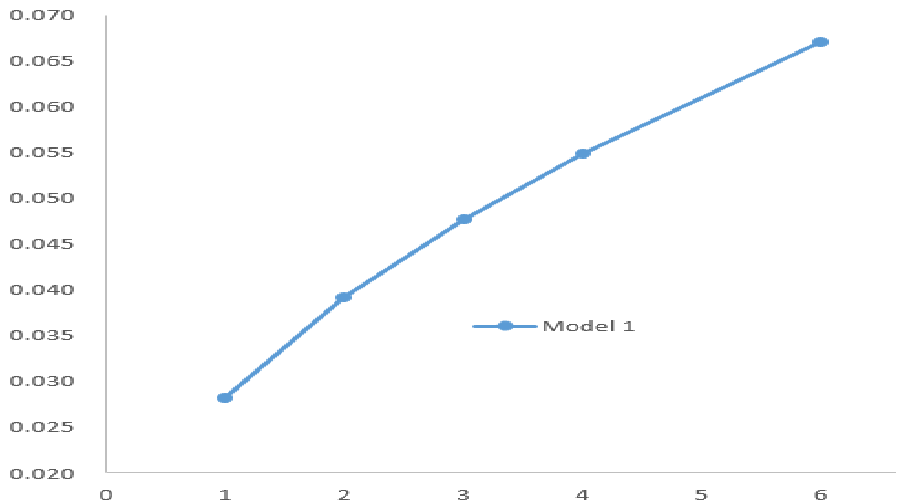
The risk free rate has been non-constant through 1960s to 2018, reaching a peak of 13% in the 1970s to near zero in the mid 2010s. The 3-month T-bill is used as a proxy for the risk-free rates. This necessitates the modelling of the risk-free rates in the original CC model which has held the log risk-free rate constant at 0.94.

Figure 4: Plot of risk-free rate against the lagged sentiment of order 2



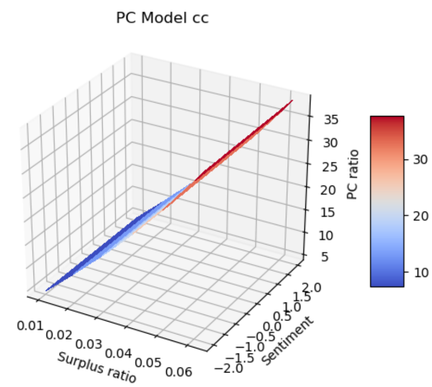
The risk free rate shows a distinct linear relationship with the lagged sentiment of order 2 which needs to be factored into the model. The paper findings are that this relationship arises primarily from the precautionary savings motive of agents and the intertemporal substitution motive.

Figure 5: Plot of percentage change in the equilibrium surplus level against risk aversion γ

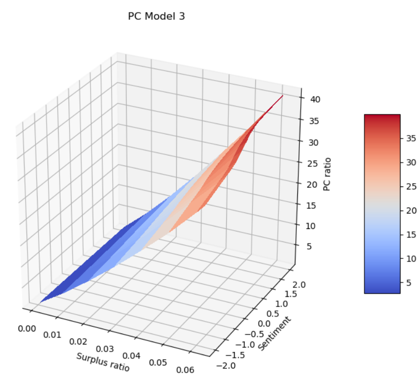


The figure plots the equilibrium surplus ratio for Model 1 and the CC model when sentiment is factored into the sensitivities $\lambda(\cdot)$ with increasing risk aversion. Note in this case, with increasing risk aversion, the equilibrium surplus ratio increases and the equilibrium habit levels decrease. As an agent becomes more risk averse, his habit expectations also decrease.

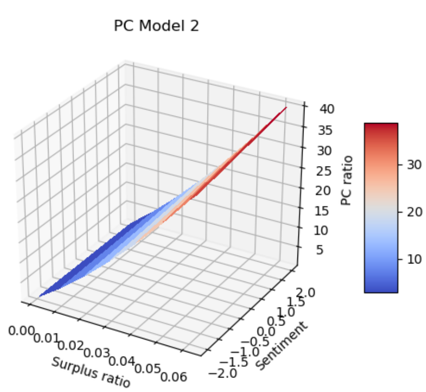
Figure 6: Price Consumption surfaces for all surfaces



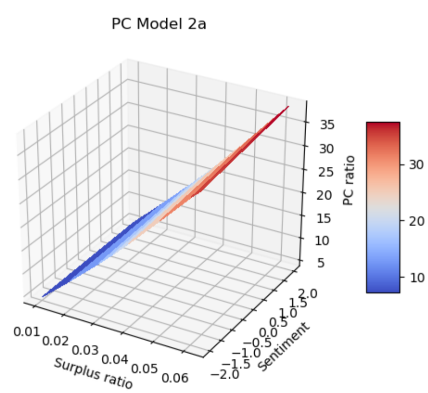
(a) CC Model



(b) Model 1



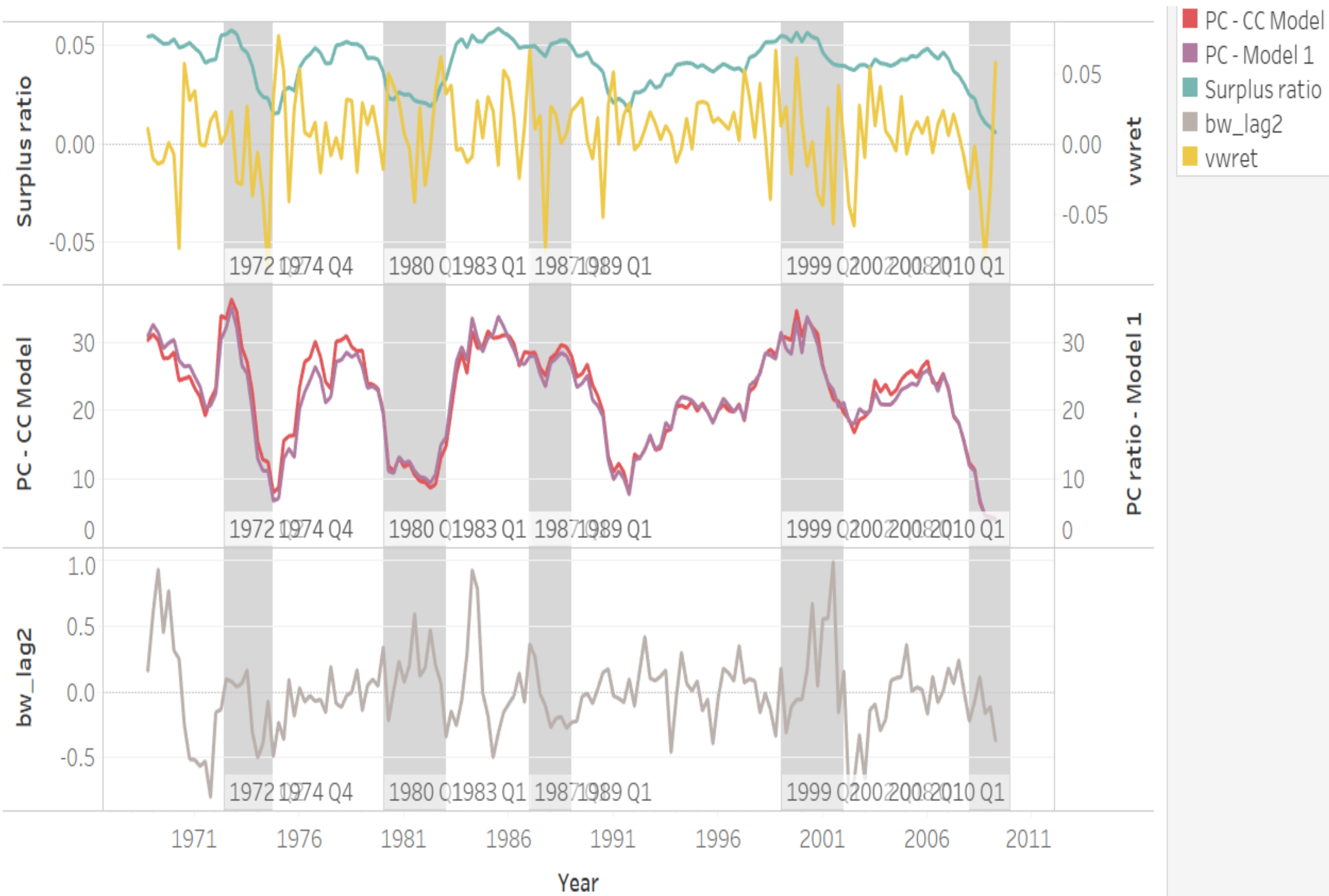
(c) Model 2



(d) Model 3

The plots show the price consumption surfaces generated by solving a fixed point solution for all the models - CC, 1, 2 and 3. There are a couple of points to note in these surfaces. These are the steepness of the surface relative to the surplus ratio and the downward tilt of negative sentiment. In this case, the Model 1 has a steeper tilt against Models 2 and 3 due to the lagged sentiment in the sensitivity function $\lambda(\cdot)$.

Figure 7: Simulated surplus ratio and PD ratios



The plots show the surplus ratios generated by the CC model that reflect the fundamentals (surplus ratios) in the economy and the PC ratios by Model 1 and CC model. Both the models PC ratios move close to each other, highlighting the surplus ratio as the more important driver of macro-economic risk compared to market sentiment. Notice the pro-cyclicity of the *lagged* market sentiment with the surplus ratio. The shaded areas refer to the financial crises period - the oil crisis in the 1970s, the stagflation in the 1980s, Black Monday in 1987, the Dot-com bust in the late 1990s/ early 2000s and the Lehman crisis in the 2008.

5.0.1 Derivation of the risk-free rate

The moments of the stochastic discount factor are derived as follows with the definitions of s_t and c_t with a time-dependent drift on the lagged sentiment in Model 1:

$$M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\frac{S_{t+1}}{S_t}\right)^{-\gamma}$$

$$s_{t+1} = (1-\phi)\bar{s} + \phi_s s_t + \lambda(s_t)\epsilon_{c,t} \quad \text{for } \epsilon_{c,t} \sim N(0,\sigma_c^2) \tag{49}$$

$$\implies s_{t+1} - s_t = (1-\phi)(\bar{s} - s_t) + \lambda(s_t)\epsilon_{c,t} \tag{50}$$

$$\text{and} \quad c_{t+1} - c_t = g_c + \alpha_x \epsilon_{x,t-6} + \epsilon_{c,t} \tag{51}$$

Thence, M_{t+1} is a lognormal variable at time t with mean μ_M and standard deviation σ_M:

$$\implies ln(M_{t+1}) = ln(\delta) - \gamma[(s_{t+1} - s_t) + (c_{t+1} - c_t)] \tag{52}$$

$$= ln(\delta) - \gamma[(1-\phi_s)(\bar{s} - s_t) + \lambda(s_t)\epsilon_{c,t} + (g_c + \alpha_x \epsilon_{x,t-6} + \sigma_c \epsilon_{c,t})] \tag{53}$$

$$= \underbrace{ln(\delta) - \gamma\Big[(1-\phi_s)(\bar{s} - s_t) + (g_c + \alpha_x \epsilon_{x,t-6})}_{\mu_M} + \underbrace{(1+\lambda(s_t))\sigma_c}_{\sigma_M} \epsilon_{c,t})\Big] \tag{54}$$

A lognormal M with mean μ and standard deviation σ has its first moment as $\mathbb{E}[M] = e^{\mu+\sigma^2/2}$. Note in this case that at time t, $\epsilon_{x,t-6}$ is known and taken out of the expectation to return the log risk-free rate:

$$\mathbb{E}_t[M_{t+1}] = \mathbb{E}_t[e^{\mu_M + \sigma_M \epsilon_{c,t}}] \tag{55}$$

$$= e^{\mu_M + \frac{\sigma_M^2}{2}} \tag{56}$$

$$\implies r_{f,t} = \frac{1}{\mathbb{E}_t[M_{t+1}]} = e^{-(\mu_M + \frac{\sigma_M^2}{2})} \tag{57}$$

By Taylor's expansion, this becomes the risk-free rate in Eq.18 in the paper below:

$$
\begin{aligned}
log(r_{f,t}) &= -(\mu_M + \frac{\sigma_M^2}{2}) && (58)\\
&= -log(\delta) + \gamma g_c + \gamma\alpha_x\epsilon_{x,t-6} - \gamma(1-\phi_s)(s_t - \bar{s}) - \frac{\gamma^2\sigma_c^2}{2}[1+\lambda(s_t)]^2 && (59)
\end{aligned}
$$

In Model 1, the $\lambda(s_t)$ and $\overline{S}$ are the same as the CC model and are:

$$
\begin{aligned}
\lambda(s_t) &= \frac{1}{\overline{S}}\sqrt{1-2(s_t-\bar{s})} - 1 \quad for\ s_t < s_{max} && (60)\\
\overline{S} &= \sigma_c\sqrt{\frac{\gamma}{1-\phi_s}} && (61)
\end{aligned}
$$

Substituting into equation 59, the risk free rate becomes a function of the lagged sentiment value.

$$
log(r_{f,t}) = -log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi_s) + \gamma\alpha_x\epsilon_{x,t-6} \tag{62}
$$

In Model 2, the $\overline{S}$ is specified as:

$$
\overline{S} = \sigma_c\sqrt{\frac{\gamma}{1-\phi_s-(B_1/\gamma)}} \tag{63}
$$

This is substituted into the below to return the risk free rate r_f that is dependent on s_t and the lagged sentiment. From the last term in Eq. 59:

$$
\begin{aligned}
\frac{\gamma^2\sigma_c^2}{2}(1+\lambda(s_t))^2 &= \frac{\gamma^2\sigma_c^2}{2}\left[\frac{1}{\overline{S}}\sqrt{1-2(s_t-\bar{s})}\right]^2 && (64)\\
&= \frac{\gamma}{2}(1-\phi-(B_1/\gamma))(1-2(s_t-\bar{s})) && (65)\\
\implies log(r_{f,t}) &= -log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi_s) + \gamma\alpha_x\epsilon_{x,t-6} + \frac{B_1}{2} - B_1(s_t-\bar{s}) && (66)
\end{aligned}
$$

In Model 1, the additional change is $\lambda(\cdot)$ specified as:

$$\lambda(S_t, \epsilon_{x,t}) = \frac{1}{\bar{S}}\sqrt{1 - 2(s_t - \bar{s}) - B_2\epsilon_{x,t-n}} - 1 \tag{67}$$

This is substituted into the below to return the risk free rate r_f that is dependent on s_t and the lagged sentiment:

$$\begin{aligned}
&\frac{\gamma^2\sigma_c^2}{2}(1 + \lambda(S_t, \epsilon_{x,t}))^2 \\
&= \frac{\gamma^2\sigma_c^2}{2}\left[\frac{1}{\bar{S}}\sqrt{1 - 2(s_t - \bar{s}) - B_2\epsilon_{x,t-2}}\right]^2 && (68)\\
&= \frac{\gamma}{2}\Big[(1 - \phi_s) - (B_1/\gamma)\Big]\Big[(1 - 2(s_t - \bar{s})) - B_2\epsilon_{x,t-2}\Big] && (69)\\
&= \frac{\gamma}{2}(1 - \phi_s) - \cancel{\gamma(1 - \phi_s)(s_t - \bar{s})} - \frac{\gamma}{2}B_2(1 - \phi_s)\epsilon_{x,t-2} - \frac{B_1}{2} + B_1(s_t - \bar{s}) + \frac{B_2 B_1}{2}\epsilon_{x,t-2} && (70)
\end{aligned}$$

This is substituted into Eq. 59 with a term cancelling out giving Eq. 22 in the paper:

$$\begin{aligned}
r_{f,t} &= -log(\delta) + \gamma g_c + \gamma\alpha_x\epsilon_{x,t-6} - \cancel{\gamma(1 - \phi_s)(s_t - \bar{s})} - \frac{\gamma^2\sigma_c^2}{2}[1 + \lambda(s_t)]^2 && (71)\\
&= -log(\delta) + \gamma g_c - \frac{\gamma}{2}(1 - \phi_s) + \gamma\alpha_x\epsilon_{x,t-6} + \frac{B_1}{2} - B_1(s_t - \bar{s}) + \frac{B_2}{2}\Big[\gamma(1 - \phi_s) - B_1\Big]\epsilon_{x,t-2} && (72)
\end{aligned}$$

5.0.2 Derivation of the conditional Sharpe ratios

Continuing from Eq. 54 for the stochastic discount factor M_t

$$\begin{aligned}
By\ definition, \quad M_{t+1} =& e^{\mu_M + \sigma_M \epsilon_{c,t}} && (73)\\
\& \quad M_{t+1}^2 =& e^{2\mu_M + 2\sigma_M \epsilon_{c,t}} && (74)\\
\implies \mathbb{E}[M_{t+1}] =& e^{\mu_M + \frac{\sigma_M^2}{2}} \quad \& \quad \mathbb{E}^2[M_{t+1}] = e^{2\mu_M + \sigma_M^2} && (75)\\
\implies \mathbb{E}[M_{t+1}^2] =& e^{2\mu_M + 2\sigma_M^2} && (76)\\
By\ definition, \quad \sigma^2[M_{t+1}] =& \mathbb{E}[M_{t+1}^2] - \mathbb{E}^2[M_{t+1}] && (77)\\
=& e^{2\mu_M + \sigma_M^2}(e^{\sigma_M^2} - 1) && (78)
\end{aligned}$$

The max Sharpe ratio is therefore:

$$max \quad \frac{\sigma(M_{t+1})}{\mathbb{E}[M_{t+1}]} = \sqrt{e^{\sigma_M^2} - 1} \tag{79}$$

The Sharpe ratio is thence only dependent on the second moment of the stochastic discount factor $\sigma_{M_t}^2$. This is expressed from Eq.54 with the different model $\lambda(\cdot)$ in table 8.

$$\sigma_{M_t}^2 = (1 + \lambda(\cdot))\sigma_c \tag{80}$$

Attribute	CC Model	Model 3	Model 2	Model 1
Consumption growth	$g_c + \epsilon_{c,t}$	$g_c + \alpha_c \epsilon_{x,t-n} + \epsilon_{c,t}$	$g_c + \alpha_c \epsilon_{x,t-n} + \epsilon_{c,t}$	$g_c + \alpha_c \epsilon_{x,t-n} + \epsilon_{c,t}$
Dividend growth	$g_c + \epsilon_{c,t}$	$g_d + \alpha_c \epsilon_{x,t-n} + \epsilon_{d,t}$	$g_d + \alpha_d \epsilon_{x,t-n} + \epsilon_{d,t}$	$g_d + \alpha_d \epsilon_{x,t-n} + \epsilon_{d,t}$
Surplus ratio process	$(1-\phi)\overline{s} + \phi s_t + \lambda(s_t)\epsilon_{c,t}$	$(1-\phi)\overline{s} + \phi s_t + \lambda(s_t)\epsilon_{c,t}$	$(1-\phi)\overline{s} + \phi s_t + \lambda(s_t)\epsilon_{c,t}$	$(1-\phi)\overline{s} + \phi s_t + \lambda(s_t, \epsilon_{x,t-n})\epsilon_{c,t}$
Log risk-free rate	$-log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi_s)$ constant at 0.94	$-log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi) + \gamma \alpha_c \epsilon_{x,t-n}$	$-log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi) + \gamma \alpha_c \epsilon_{x,t-n} + \frac{B_1}{2} - B_1(s_t - \overline{s})$	$-log(\delta) + \gamma g_c - \frac{\gamma}{2}(1-\phi) + \gamma \alpha_c \epsilon_{x,t-n} + \frac{B_1}{2} - B_1(s_t - \overline{s}) + \frac{B_2}{2}[\gamma(1-\phi_s) - B_1]\epsilon_{x,t-n}$
$\overline{S}$	$\sigma_c \sqrt{\frac{\gamma}{1-\phi}}$	$\sigma_c \sqrt{\frac{\gamma}{1-\phi}}$	$\sigma_c \sqrt{\frac{\gamma}{1-\phi-(B_1/\gamma)}}$	$\sigma_c \sqrt{\frac{\gamma}{1-\phi-(B_1/\gamma)}}$
$\lambda(\cdot)$	$\frac{1}{\overline{S}}\sqrt{1-2(s_t-\overline{s})} - 1$	$\frac{1}{\overline{S}}\sqrt{1-2(s_t-\overline{s})} - 1$	$\frac{1}{\overline{S}}\sqrt{1-2(s_t-\overline{s})} - 1$	$\frac{1}{\overline{S}}\sqrt{1-2(s_t-\overline{s}) - B_2\epsilon_{x,t-n}} - 1$
s_{max}	$\overline{s} + \frac{1}{2}(1-\overline{S}^2)$	$\overline{s} + \frac{1}{2}(1-\overline{S}^2)$	$\overline{s} + \frac{1}{2}(1-\overline{S}^2)$	$\overline{s} + \frac{1}{2}(1-\overline{S}^2 - B_2\epsilon_{x,t-n})$

The table is a summary of the key attributes across the different models - CC, Model 1, 2 and 3. The Models 1, 2 and 3 all differ from the CC model with consumption and dividend drifts that are dependent on the lagged sentiment shock. In addition, Model 2 considers the surplus ratio in the risk-free rate. The Model 1 adds on lagged sentiment shock into the prudence measure. This allows to model the risk-free rate that in the CC model is set to be a constant at 0.94 (the log risk free rate), which is restrictive.

Table 8: Summary of Habit / Sentiment Models